MATHEMATICAL INDUCTION

OBJECTIVE PROBLEMS

1. **For all** $n \in N, 3.5^{2n+1} + 2^{3n+1}$ **is divisible by**

   1) 19 2) 17

   3) 23 4) Any odd integer

2. **For all +ve integral values of** $n, 49^n + 16n - 1$ **is divisible y**

   1) 64 2) 8 3) 16 4) 43

3. **For all** $n \in N, 10^n + 3, 4^{n+2} + 5$ **is divisible by**

   1) 23 2) 3 3) 9 4) 207

4. **The number** $a^n - b^n$ **(a, b are distinct rational numbers and (** $n \in N$ **) is always divisible by**

   1) $a - b$ 2) $a + b$ 3) 2a-b 4) a-2b

5. **The number** $a^n + b^n$ **is divisible by – when n is an odd +ve integer but not when n is an even +ve integer.**

   1) 1) a-b 2) a+b 3) 2a-b 4) 2a+b

6. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + --- $ **to n terms =**

   1) 1/5n-1 2) 1/n+4 3) n/3n+1 4) n/5n-1

7. $49^n + 16n + k$ **is divisible by 64 for** $n \in N$ **. Then the numerically least -ve integral value of k is**

   1_-2 2)-1 3) -3 4) -4

8. **1.2.3+2.3.4+3.4.5+------- to n terms =**

   1) $\frac{(n+1)(n+2)(n+3)}{4}$

   2) $\frac{(n+2)(n-2)(n-3)(n+3)}{4}$

   3) $\frac{n(n+1)(n+2)(n+3)}{4}$

   4) None

9. **For** $n \in N, (1/5)n^5 + (1/3)n^3 + (7/15)n$ **is**

1) An integer 2) A natural number

3) a +ve fraction 4) None

10. The nth term of the series 4+14+30+52+80+114+----- is

1) 5n-1 2) $2n^2 + 2n$

3) $3n^2 + n$ 4) $2n^2 + 2$

11. Sum of n terms of the series $1^3 + 3^3 + 5^3 + ---$ **is**

1) $n^2(n^2 - 1)$ 2) $n^2(2n^2 - 1)$

3) $n^2(2n^2 + 1)$ 4) $n^2(2n^2 + 1)$

12. If 1+5+12+22+35+------- to n terms = $\frac{n^2(n+1)}{2}$, n^{th} **term of L.H.S. is**

1) $\frac{n(4n-1)}{3}$ 2) $\frac{n(3n-1)}{2}$

3) $\frac{n(3n+1)}{2}$ 4) $\frac{n(4n+1)}{3}$

13. 1/3.5+1/5.7+1/7.9+........ to n terms =

1) $\frac{n}{3(2n+3)}$ 2) $\frac{n}{2n+3}$

3) $\frac{1}{(n+2)(n+4)}$ 4) none

14. $1^3 + 2^3 + 3^3 + ...+ 100^3 = k^2$ **then k =**

1) 10100 2) 5000 3) 5050 4) 1010

15. $10^{2n-1} + 1$ **for all** $n \in N$ **is divisible by**

1) 2 2) 3 3) 7 4) 11

16. Then n^{th} **term of the series 3+7+13+21+..... is**

1) 4n-1 2) $n^2 + 2n$

3) $n^2 + n + 1$ 4) $n^2 + 2$

17. 2.3+3.4+4.5+........ to n terms =

1) $\frac{n(n^2+6n+14)}{9}$ 2) $\frac{n(n^2-6n+11)}{6}$ 3) $\frac{n(n^2+6n+11)}{3}$ 4) None

18. $(1^2+1)+(2^2+2)+(3^2+3)+...+(n^2+n)=$

1) $\frac{(n+1)(n+2)}{3}$ 2) $\frac{n(n+1)(n+2)}{3}$

3) $\frac{n(n+1)(n+2)}{2}$ 4) None

19. **1+3+7+15+....... to n terms =**

1) $2^{n+1}+n-2$ 2) n^2+n-2

3) 2^n-1 4) None

20. **2+8+14+..... to n terms =**

1) 3n-1 2) 4n-2

3) $3n^2-n$ 4) $\frac{2(4n^2-1)}{3}$

21. **For all odd positive integers n, number** $n(n^2-1)$ **is divisible by**

1) 3 2) 4 3) 6 4) 24

22. **1.3+3.5+5.7+......+ to n term =**

1) $\frac{n(4n^2+6n-1)}{3}$ 2) $\frac{4n^2+6n-1}{3}$

3) n^2+n+1 4) None

23. **If the sum to n terms of an A.P is** $\frac{4n^2-3n}{4}$**, then the nth term of the A.P. is**

1) $\frac{5n-1}{4}$ 2) $\frac{8n-7}{4}$ 3) $\frac{3n^2-2}{4}$ 4) None

24. **If 3+5+9+17+33+........ to n terms =** $2^{n+1}+n-2$**, then nth term of L.H.S. is**

1) 3^n-1 2) $2n+1$ 3) 2^n+1 4) 3n-1

25. **If** $1+\frac{1}{1+2}+\frac{1}{1+2+3}+....+\frac{1}{1+2+....+n}=\frac{Kn}{n+1}$ **for all** $n\in N$**, then K =**

1) 1 2) 2 3) 3 4) ½

26. **For** $n\in N$**,** $\cos\alpha, \cos 2\alpha, \cos 4\alpha \; ......\cos 2^{n-1}\alpha =$

1) $\frac{\sin 2^n\alpha}{2\sin\alpha}$ 2) $\frac{\sin 2^n\alpha}{2\cos\alpha}$ 3) $\frac{\sin 2^n\alpha}{2\sin\alpha}$ 4) $\frac{\cos 2^n\alpha}{2^n\cos\alpha}$

27. $7^{2n}+3^{n-1}.2^{3n-3}$ **is divisible by**

1) 24 2) 25 3) 9 4) 13

28. If $n\in N$ **and 1.3+3.5+5.7+....+(2n-1)(2n+1) =** $\frac{4n^3+6n^2-n}{K}$**, then K =**

1)1 2) 2 3) 3 4) 5

29. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)......\left(1-\frac{1}{n}\right)$ $(n\ge 3)=$

1) $\frac{1}{n^2}$ 2) $\frac{1}{n^3}$ 3) $\frac{2}{n}$ 4) $\frac{1}{n}$

30. If $n\in N$**. Then** $n\left(n^2-1\right)$ **is divisible by**

1) 6 2) 16 3) 36 4) 24

31. The product of υ **consecutive integers is divisible by**

1) n 2) n^n 3) n! 4) (n-1)

32. If P(n) is a statement such that truth of P(n) $\Rightarrow$ **the truth of P(n+1) for** $n\in N$**, then P(n) is true**

1) $\forall n$

2) For all n>1

3) For all n >m, m is some fixed positive integer

4) Nothing can be said

33. If P(n) :2n<n !, n$\in N$**, then P(n) is true for**

1) All n 2) all n > 2 3) all n > 3 4) None

34. A student was asked to prove a statement by induction. He proved (i) P(5) is true and (iii) truth of P(n) $\Rightarrow$ **truth of P(n+1),** $n\in N$**. On the basis of this, he could conclude that P(n) is true.**

1) For no n 2) For all $n\ge 5$

3) For all n 4) None of these

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MATHEMATICAL INDUCTION

HINTS AND SOLUTIONS

1. (2)

When $n = 1, 3 \cdot 5^{2n+1} + 2^{3n+1} = 3 \times 125 + 16 = 391$

$n = 2, 3.5^{2n+1} + 2^{3n+1} = 9375 + 128 = 9503$

H.C.F. of 391, 9503, … is 17.

2. (1)

When $n = 1, 49^n + 16n - 1$ = 49 + 16 – 1= 64

$n = 2$, $49^n + 16n - 1 = 49^2 + 16 \times 2 - 1$

= 2401 + 32 – 1 = 2432, ……

3. (3)

When n = 1, $10^n + 3.4^{n+2} + 5 = 207$,

$n = 2, 10^2 + 3(4)^4 + 5$

= 100 + 768 + 5 = 873 ……

HCF of 207, 873 is 9.

4. (1)

$f(a) = a^n - b^n, n \in N \Rightarrow f(b) = (b)^n - b^n = 0$

$\Rightarrow a^n - b^n$ is divisible by a – b.

5. (1)

$f(a) = a^n + b^n, n \in N$

$\Rightarrow f(-b) = (-b)^n + b^n = 0$ When n is an odd +ve integer and not equal to zero when n is an even +ve integer.

$\Rightarrow$ divisible by a + b

6. (2)

$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \ldots$ to n terms

$$= \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \ldots + \frac{1}{(3n-2)(3n+1)}$$

$$= \sum \left[\frac{1/3}{3n-2} - \frac{1/3}{3n+1} \right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \ldots\ldots\ldots + \left(\frac{1}{3n-2} - \frac{1}{3n+1}\right) \right]$$

$$= \frac{1}{3}\left[1 - \frac{1}{3n+1}\right] = \frac{n}{3n+1}.$$

7. (3)

$49^n + 16n + k$ is divisible by 64 and k is the least –ve integer.

$\Rightarrow$ 49 + 16 + k is divisible by 64

$\Rightarrow$ 65 + k is divisible by 64

$\Rightarrow$ k = –1.

8. (2)

$S_n = 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + 3\cdot 4\cdot 5 + \ldots$ to n terms

Put n = 1, $S_1 = 1\cdot 2\cdot 3 = 6$

Put n = 2, $S_2 = 6 + 24 = 30$

When n = 1,

$\frac{n(n+1)(n+2)(n+3)}{4} = \frac{1\cdot 2\cdot 3\cdot 4}{4} = 6 = S_1$ and

When n = 2, $\frac{n(n+1)(n+2)(n+3)}{4} = \frac{2\cdot 3\cdot 4\cdot 5}{4} = 30 = S_2$.

9. (2)

For $n \in N$, $P(n) = \frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$

$\Rightarrow P(1) = 1/5 + 1/3 + 7/15 = 1,$

$P(2) = 32/5 + 8/3 + 14/15$

$= \frac{96+40+14}{15} = 10, \ldots\ldots$

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10. (3)

$t_n = n^{th}$ term $= 3n^2 + n$

When $n = 1 \Rightarrow t_n = 4$,

When $n = 2, \Rightarrow t_n = 14$,

$n = 3, \Rightarrow t_n = 30$ etc.

11. (2)

In(2) : $S_1 = 1^2\,(2.1^2 - 1) = 1 = 1^3$

$S_2 = 2^2\,(2.2^2 - 1) = 28 = 1^3 + 3^3$.

12. (2)

$$S_n = \frac{n^2(n+1)}{2}$$

$$S_{n-1} = \frac{(n-1)^2(n-1+1)}{2} = \frac{(n-1)^2 n}{2}$$

$$\Rightarrow \text{nth term} = S_n - S_{n-1}$$

$$= \frac{(n-1)^2 n}{2} - \frac{(n-1)^2(n)}{2}$$

$$= \frac{n[n^2 + n - n^2 + 2n - 1]}{2}$$

$$= \frac{n(3n+1)}{2}.$$

13. (1)

$$\text{In(i)}: S_1 = \frac{1}{3(2\cdot 1+3)} = \frac{1}{3.5}$$

$$S_2 = \frac{2}{3(2\cdot 2+3)} = \frac{2}{3\cdot 7}$$

$$= \frac{7+3}{3\cdot 5\cdot 5} = \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7}.$$

14. (3)

$$1^3+2^3+3^3+...+100^3=\sum_{n=1}^{100} n^2=\left(\frac{n(n+1)}{2}\right)^2 \text{ where n = 100.}$$

$$=\left(\frac{100\times101}{2}\right)^2=(5050)^2=K^2$$

$$\Rightarrow K=5050$$

15. (4)

$10^{2n-1}+1, n=1\Rightarrow 11$

$n=2\Rightarrow 1001...$

$\therefore 10^{2n-1}+1$ is divisible by 11.

$\therefore$ HCF of 11, 1001 is 11.

16. (3)

In(3) n^2+n+1, put $n = 1 \Rightarrow 3$

Put $n = 2 \Rightarrow 7$……

17. (3)

Take $\frac{n(n^2+6n+11)}{3}$, n=1 $\Rightarrow \frac{18}{3}=6=2\cdot3$

$n=2\Rightarrow \frac{2(27)}{3}=2.3+3.4$

$n=3\Rightarrow 38=2.3+3.4+4.5$ etc.

18. (2)

nth term = n^2+n

$$\therefore S_n=\sum(n^2+n)=\sum n^2+\sum n$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right]=\frac{n(n+1)(n+2)}{3}.$$

19. (4)

$S_n = 1+3+7+15+\ldots$ to n terms

$\Rightarrow S_1 = 1, S_2 = 1+3 = 4, S_3 = 1+3+7 = 11, \ldots$

n = 1 $\Rightarrow$ (1) is not true.

n = 2 $\Rightarrow$ (2) is true, (3) is not true.

n = 3 $\Rightarrow n^2 + n - 2 = 9 + 3 - 2 = 10 \neq S_2$

$\Rightarrow$ (2) is not true.

20. (3)

$3n^2 - n$

n = 1 $\Rightarrow$ 2

n = 2 $\Rightarrow$ 10, …

21. (4)

$n(n^2 - 1)$, n = 3 $\Rightarrow$ 3(8) = 24

n = 5 $\Rightarrow$ 5(24), n = 7 $\Rightarrow$ 7(48)…

$\Rightarrow$ Divisible by 24

22. (1)

$\frac{n(4n^2+6n-1)}{3}$, n = 1 $\Rightarrow$ 1.3 = s_1

n = 2 $\Rightarrow$ 18 = 1.3 + 3.5 = s_2

23. (2)

$S_n = \frac{4n^2-3n}{4}$,

$t_n = n^{th}$ term = $S_n - S_{n-1}$

$= \frac{4n^2-3n}{4} - \frac{4(n-1)^2-3(n-1)}{4}$

$= \frac{1}{4}^{[4\{n^2-(n-1)^2\}-3\{n-(n-1)\}]} = \frac{1}{4}^{(8n-7)}$

24. (3)

$S_n = 2n + 1 + n - 2$

$S_n - 1 = 2n + n - 1 - 2$

$\Rightarrow \text{nth term} = S_n - S_{n-1}$

$= 2 \cdot 2^n + n - 2 - 2^n - n + 1 + 2 = 2^n + 1.$

25. (2)

Put n = 2.

Then $1 + \frac{1}{1+2} = \frac{2K}{3} \Rightarrow K = 2$

26. (3)

$$\cos\alpha \cdot \cos 2^1\alpha \cdot \cos 2^2\alpha ......\cos 2^{n-1}\alpha = \frac{1}{2\sin\alpha} \cdot \sin 2^1\alpha \cdot \cos 2^1\alpha \cdot \cos 2^2\alpha ...\cos 2^{n-1}\alpha$$

$$= \frac{1}{2^2\sin\alpha}(2\sin 2^2\alpha \cdot \cos 2^2\alpha ...\cos 2^{n-1}\alpha$$

$$= \frac{1}{2^n\sin\alpha} \cdot \sin 2^n\alpha.$$

(Or) Put n = 1.

L.H.S. $\cos 2^0\alpha = \cos\alpha$

R.H.S. = $\frac{\sin 2^1\alpha}{2\sin\alpha} = \cos\alpha$

Put n = 2.

L.H.S. $\cos\alpha \cdot \cos 2^1\alpha$

R.H.S. $= \frac{\sin 2^2\alpha}{2^2\sin\alpha} = \frac{2\sin 2\alpha \cdot \cos 2\alpha}{4\sin\alpha}$

$$= 4\frac{\sin\alpha \cdot \cos\alpha \cdot \cos 2\alpha}{4\sin\alpha} = \cos\alpha \cdot \cos^2\alpha$$

27. (2)

n = 1 G.E. $7^2 + 3^0 \,.\, 2^0 = 50$

n = 2 G.E. = $7^4 + 3.2^3 = 2425$

G.C.D of 50, 2425 is 25.

28. (2)

$$n = 2 : 1.3 + 3.5 = \frac{32 + 24 - 2}{K}$$

$$\Rightarrow 18 = \frac{54}{K} \Rightarrow K = 3$$

29. (4)

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)...\left(1 - \frac{1}{n}\right)$$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} ... \frac{n-1}{n} = \frac{1}{n}$$

30. (1)

$n(n^2 - 1) = (n - 1)n(n + 1)$

= products of 3 consecutive integers divisible by $\underline{|3} = 6$.

31. (3)

The product of three consecutive integers are (n – 1) n (n + 1), it is divisible by ∠3.

The product of four consecutive integers are (n – 1) n (n + 1)(n + 2), it is divisible by ∠4.

⇒ the product of n consecutive integers is divisible by ∠n.

32. (4)

We cannot set anything above the truth of P(n), ∀n ∈ N since truth of P(1) is not given.

33. (3)

P(1), P(2), P(3) are not true.

P(4) is true. Also, $2^m < \angle m$

$$\Rightarrow 2 \cdot 2^m < 2 \cdot \underline{|m}$$

$$\Rightarrow 2^{m+1} \le (m+1) \cdot \underline{|m} \text{ for } m \ge 1$$

$$\Rightarrow 2^{m+1} \ge \underline{|(m+1)}, \text{for } m \le 1.$$

34. (2)

By the principle of mathematical induction P (n) is true for all n ≥ 5.

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